

# An Experimental Methodology to Characterise Post-Necking Behaviour and Quantify Ductile Damage Accumulation in Isotropic Materials

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## Abstract

The development of ductile damage, that occurs beyond the point of necking in a tensile test, can be difficult to quantify. An experimental methodology has been developed to accurately characterise the post-necking deformation response of a material through continuous monitoring of the specimen's shape up until rupture. By studying the evolution of the neck geometry, the correct values of the local stress and strain have been determined in samples of grade 304L stainless steel and C110 copper. Notched bar specimens of various notch acuities were examined enabling the effects of stress triaxiality on ductile fracture to be determined. The methodology developed has provided a robust framework for macroscopic measurements of ductile damage during the necking process. To characterise the material degradation process, the elastic modulus reduction method was employed on hourglass-shaped specimens of the same materials. Stiffness degradation was measured using a small gauge extensometer during uninterrupted tensile tests with partial elastic unloadings. A metallographic study was conducted on progressively damaged specimens in order to validate the macroscopic damage measurements. A new non-linear ductile

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damage accumulation law has been developed and calibrated, which provides an advanced representation of the experimental results, and a significant improvement compared to linear accumulation models frequently employed. This realistic modelling approach considers the degradation of the material when it has undergone severe plastic deformation, and provides a more accurate representation of the near failure behaviour by considering the effects of stress triaxiality. The methodology provides accurate data for damage model development and calibration, to improve the predictions of remnant life from ductile damage in engineering components.

*Keywords:* Plasticity, Necking, Triaxiality, Ductile damage, Ductile fracture

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## 1. Introduction

The ability to characterise very high strain deformation behaviour, and to quantify and predict ductile damage accumulation in a material is important for many engineering applications. For example, automotive industry designs its vehicles so that their bodywork plastically deforms, absorbing the energy of an impact. Components manufactured using metal forming experience very high plastic deformation, whilst avoiding failure by ductile damage. Hence models to describe the ductile damage accumulation to very high strains are required. Modern experimental and simulation techniques are capable of representing severe plasticity and material degradation, however, most constitutive laws employed in these models are obtained by conventional instrumentation, assuming uniform deformation through the sample, and thus limited to the stress-strain values at the point of strain localisation by necking.

The final stages of localised plastic deformation in the form of a neck in ductile isotropic materials were studied on round specimens by Bridgman (1964), who derived relationships between the geometry of the neck and the stresses and strains generated. These local stresses and strains represent the actual plastic behaviour of the material compared to a classical uniform tensile test approximation. With modern high resolution cameras, it is now possible to measure

20 and track accurately the curvature of the material in the necked region. High  
 resolution imaging of the neck geometry combined with Bridgman (1964) the-  
 ory, or a similar necking approximation, makes it possible to derive the stresses  
 and strains in the centre of the neck, and hence the hardening behaviour to  
 high strains. Rosa et al. (2003) performed studies on the neck development  
 25 of metallic materials investigating alternative methods of neck characterisation  
 without the need of geometry monitoring. Mirone (2004) further developed that  
 approach proposing a material-independent necking model which required only  
 the experimental true stress-strain curve and the neck onset strain to calculate  
 the equivalent stress and equivalent strain. This analytical approach could be  
 30 used as an alternative to the Bridgman model without the need of experimen-  
 tal profile curvature measurements. Other alternative approaches to consider  
 the necking behaviour include the use of the virtual fields method to calculate  
 post-necking hardening proposed by Kim et al. (2013); or the inverse methods  
 based on Finite Element Model Updating (FEMU) (e.g. Kajberg and Lind-  
 35 kvist (2004); Koc and Stok (2004)). This technique consists of deducing the  
 parameters of the model from its experimentally obtained solution. In the case  
 of material necking, stain fields in the specimen surface can be measured using  
 Digital Image Correlation (DIC), and finite elements simulations can be solved  
 iteratively updating the model until the difference between the measured and  
 40 estimated strain fields is minimised (Kajberg and Lindkvist (2004)).

Focusing on the methods that use the experimental measurement of the  
 sample's profile, such as the Bridgman approach, several techniques have been  
 proposed to record the shape of the specimen: G'Sell et al. (1992) used cameras  
 to record the necking in polymers while Gerbig et al. (2016) opted for using  
 45 Digital Image Correlation. Rittel et al. (2014) used high-speed cameras in Split  
 Hopkinson Tensile Bar experiments to observe the necking of metals in dynamic  
 conditions and Vaz-Romero et al. (2016) followed a similar approach to investi-  
 gate the effect of multiaxiality in dynamic necks development. The expanding  
 ring experiment is an alternative technique that can be used to quantify necking  
 50 at even higher strain-rates ( $\dot{\epsilon} \sim 10^{-4} s^{-1}$ ), Zhang and Ravi-Chandar (2006).

The methodology proposed here makes use of Bridgman’s approach but introduces a simplified method of neck profile measurement that requires just a single DSLR camera (Digital Single-Lens Reflex), and uses an automated image processing algorithm for neck extraction; therefore, it provides an effective and economical method to accurately characterise the necking phenomenon experimentally. The results obtained can be used to calibrate popular plasticity models such as Ramberg-Osgood (Ramberg and Osgood (1943)) and Johnson-Cook (Johnson and Cook (1983)), in addition to more complex models such as the mechanical threshold stress (MTS) model, which is based on dislocation theory (Mecking and Kocks (1981)), or the Bai and Wierzbicki (2008) model, which accounts for the effect of hydrostatic pressure and deviatoric stress.

The ability to characterise the shape of the neck allows the effects of stress triaxiality on the fracture behaviour of the material to be determined. Tensile tests on notched bars of different notched acuities are generally employed (see e.g. Bai and Wierzbicki (2008); Brünig et al. (2008)), to determine stress triaxiality effects on ductility. Stress triaxiality changes during the deformation process, however, when calibrating fracture models, the tensile ductility is associated with a single stress triaxiality ratio, which is effectively assumed constant during the test. In the past, the initial or final value of stress triaxiality was used for that purpose (Johnson and Cook (1985)). In more recent studies, the full history of the stress state obtained from finite elements simulations has generally been considered, e.g. Bai and Wierzbicki (2008). Considerable efforts have been made recently in an attempt to design ductile fracture experiments that produce locally proportional loading histories. In the methodology proposed here, the evolution of stress triaxiality along the test can be obtained experimentally. This is achieved by tracing the geometry of the neck, enabling the influence of stress state on ductility to be characterised, and enhancing the accuracy of ductile fracture models. Another parameter that is known to affect ductility of metals is Lode angle, which is related to the second and third invariants of the deviatoric stress tensor, and represents the type of loading that the material is undergoing (tension, compression, shear or a combination e.g.

mixed-mode loading). A wide variety of specimen geometries and experimental configurations have been proposed in order to study different combinations of stress triaxiality and Lode angle, e.g. cruciform and butterfly specimens, which  
85 were firstly proposed by Mohr and Henn (2007), and later used by other authors such as Xue and Wierzbicki (2008). However, the research presented here is focused on the study of stress triaxiality effects and Lode angle effects have not been analysed, therefore, only stress states similar to the ones studied can be represented at present by the methodology discussed in this paper.

90 Ductile damage accumulation rate, a measure of the plastic degradation of the material with plastic strain, has proven difficult to determine experimentally (Lemaitre and Dufailly (1987), Alves (2001)). Hence, very often a linear accumulation of damage rate is assumed. Various techniques have previously been proposed to calculate damage accumulation, all based on measuring  
95 a macroscopic mechanical property that is considered to be directly related to damage. The indirect methods include elastic modulus reduction measurements (Lemaitre and Dufailly (1987); Bonora et al. (2011)), hardness measurements (Oliver and Pharr (2004); Guelorget et al. (2007)), micro-structure analysis (Lemaitre and Dufailly (1987)), response to ultrasonic waves (Boccaccini and  
100 Boccaccini (1997); Kumar et al. (2010)), X-Ray micro-tomography (Cao et al. (2014)) and electric potential drop (Zhang et al. (2014)). In this work, an advanced stiffness reduction technique has been developed to measure damage. This features the use of a small extensometer, which combined with the aforementioned necking correction technique provides a fast and effective technique for damage quantification. The results obtained using this technique have  
105 been validated by observation of voids density from microstructural images of material that had undergone different levels of plastic deformation. An alternative approach to investigate damage accumulation rate would be to induce non-proportional paths during testing, and tune the adopted damage model  
110 according to the differences in terms of strain to fracture observed.

An aim of this study is to prove that damage accumulation is highly non-linear for most materials, as previously presented by Bonora et al. (2011). Dif-

ferent approaches exist to represent ductile damage and there are different opinions about its effect on material behaviour. Micro-mechanical models such as GNT (Gurson-Needleman-Tvergaard) (Needleman and Tvergaard (1984)) represent the actual processes of void nucleation, growth and coalescence involved in ductile damage. Continuum damage mechanics models like Bonora (1997) consider damage through its effects on macroscopic properties. These two approaches introduce a coupled modelling of damage, and include elastic and/or plastic softening of the material with ductile damage. Phenomenological models such as Johnson and Cook (1985) and Bai and Wierzbicki (2008) fit the fracture strain conditions to analytical expressions and generally assume a linear damage accumulation with no effect of damage on the elasto-plastic behaviour. A new, non-linear damage accumulation law is proposed here, which is based on the model proposed by Cortese et al. (2016) for non-proportional loading conditions, and also takes concepts from the continuum damage mechanics theory. This model can be used in simulations to calculate the level of material degradation, and can be coupled with the elasto-plastic calculations for a more realistic material representation. This new modelling approach has been carefully validated with appropriate experimentation.

## 2. Methodology

The materials and specimen geometries employed in this work are described below. The experimental methodologies developed for plasticity and damage assessment are also presented and the material models analysed. All the experiments discussed have been performed in quasi-static ( $\dot{\epsilon} \sim 10^{-3} \text{ s}^{-1}$ ) and room temperature conditions ( $T \sim 20 \text{ }^{\circ}\text{C}$ ).

### 2.1. Materials

The behaviours of two different materials have been investigated, one an austenitic 304L stainless steel with low carbon (max. 0.03%) and minimum 18% chromium and 8% nickel contents; the other C110 copper, an alloy with

>99.90% copper content and oxygen generally below 0.04%. These materials have been selected due to their appreciable ductility and the availability of published work for comparison purposes.

## 2.2. Specimens

145 The test specimens were manufactured from rods in the as-drawn condition. Three different specimen geometries were used in this investigation: smooth round bars, notched round bars and hourglass shaped round bars. An image of the types of specimens tested is presented in Figure 1 and a summary of the dimensions of all the samples can be found in Table 1.

150 For plasticity and necking characterisation a conventional round bar specimen (RB) of 6 mm diameter and 30 mm gauge length as employed, as shown in Figure 2 (Test No. 1 in Table 1). For damage monitoring experiments, hourglass shaped specimens (HGR) were used (Figure 3, Test No. 2 in Table 1). The region of interest in these specimens is a central gauge region of uniform diameter  
155 6 mm and length 6 mm, where damage will be concentrated. Above and below this region of interest the sample is tapered presenting a gradual narrowing, in order to avoid stress concentrations. The equal stress and deformation behaviour of RB and HGR specimens in their gauge section was verified with FE simulations. Similar geometries were used in the past for damage monitoring  
160 (Alves et al. (2000)) and necking analysis (Gerbig et al. (2016)).

Lastly, in order to study the effect of stress triaxiality on fracture strain, round notched bar specimens (RNB) with different notch acuties were used. Equation 1 can be derived from Bridgman's theory (Bridgman, 1964) and provides the initial value of stress triaxiality in the minimum cross section of the  
165 sample as a function of the geometry of the notch:

$$\eta = \frac{\sigma_H}{\bar{\sigma}} = \frac{1}{3} + \ln \left( 1 + \frac{a}{\rho} \right) \quad (1)$$

where  $\eta$  is the stress triaxiality,  $\sigma_H$  the hydrostatic stress,  $\bar{\sigma}$  the equivalent von Mises stress,  $a$  the cross-sectional radius in the notch and  $\rho$  the radius of curvature of the notch. Different notch geometries were tested to represent

various states of stress triaxiality (Tests No. 3-8 in Table 1), an example of  
170 which is shown in Figure 4.

### 2.3. *Experimental procedure for plasticity modelling*

A conventional tensile test is usually performed to determine the elastoplastic  
behaviour of materials using an extensometer to measure elongation and a load  
cell to obtain load. The true stress - strain performance of the material can then  
175 be derived from these measurements assuming uniform deformation behaviour  
using Equations 2 and 3:

$$\varepsilon_T = \ln(1 + \varepsilon_e) \quad (2)$$

$$\sigma_T = \sigma_e(1 + \varepsilon_e) \quad (3)$$

where  $\sigma_e$  and  $\varepsilon_e$  are the engineering stress and strain, and  $\sigma_T$  and  $\varepsilon_T$  the true  
stress and strain. The values obtained following this procedure are only valid  
180 while the material deforms uniformly, but the behaviour stops being uniform  
once localisation starts.

The experimental setup employed for the post-necking behaviour measure-  
ments during tensile tests can be seen in Figure 5, which includes a DSLR camera  
and a light source on each side of the sample. The camera used was a Nikon  
185 D7000 with 16.2 megapixel DX-format CMOS image sensor. The camera was  
carefully focused using a flat sheet located at the mid thickness position where  
the specimen was later placed, ensuring focus at the mid-plane of the specimen.  
By using a relatively large aperture (f/2), low exposure time (1/1000), low ISO  
(100), a focal length of 200 mm and an approximate distance to the sample  
190 of 600 mm the depth of field obtained was 0.5 mm, which is very narrow but  
assures that a sharp representation of the specimen's profile is obtained. The  
camera setup must not be moved after it has been calibrated, since with the  
narrow depth of field used small changes in the distance to the sample would  
lead to unacceptably blurry specimen edges in the images.

195 Additional lighting enables settings on the camera to be used that optimise  
the sharpness of the image, sacrificing its brightness. They also increase the



contrast between the specimen and the background making it easier to determine accurately the external profiles from the images. The specimen was painted white to improve contrast, as well as to avoid reflections from the metallic surface. The experiments are best performed soon after the samples have been painted so that the paint is still slightly wet, such that it is more likely to remain attached to the surface of the specimen. Generally, the tests were performed 1-2 hours after the specimens had been painted.

For profile sample extraction an image thresholding method was used. With this setup, the transition between the dark background of the image and the light region of the sample presents only 2-3 pixels of thickness with varying shades of grey, as observed in Figure 6. The equivalent size of each pixel can be obtained by comparing the actual diameter of the sample with the number of pixels across the specimen before the test, and in the case represented the side of each pixel measures  $7.8 \mu\text{m}$ . If 3 pixels of dimensional uncertainty are considered for the thresholding method, the error of the profile measurement would be  $\leq 23.4 \mu\text{m}$ , which can also be taken as the accuracy of the method.

The tests were conducted under quasi-static conditions ( $\dot{\epsilon} \sim 10^{-3} \text{ s}^{-1}$ ) in a conventional 250 kN tensile Instron machine. Images were recorded at a rate of 0.5 fps. This resulted in an average of 370 images taken along the stainless steel tests and 140 images along the copper tests, approximately 110 of which images corresponded to the necking region for both materials. An automated script was created to extract the specimen external profiles of each picture after having applied the threshold to the image. From the profiles, the minimum cross-sectional radius ( $a$ ) and the fitted osculating circles radii ( $\rho$ ) can be determined, as observed in Figure 7.

The theory derived by Bridgman (1964) can be used to obtain the true stress - true strain behaviour in the central point of the necked cross-section of the specimen. That is the location of interest for the present research, since the maximum value of stress triaxiality and ductile damage appears there during the localisation process, and being the aim to characterise damage, the most demanding conditions that first lead to fracture should be analysed. By mon-

itoring the minimum radius of the sample the average axial strain in the neck can be obtained using Equation 4 and the average axial stress with Equation 5:

$$\bar{\varepsilon} \approx \varepsilon_{zz,av} = 2 \ln \left( \frac{a_0}{a} \right) \quad (4)$$

230

$$\sigma_{zz,av} = \frac{Load}{\pi a^2} \quad (5)$$

where  $\varepsilon_{zz,av}$  and  $\sigma_{zz,av}$  represent the average strain and stress across the cross-section in the specimen axial direction,  $\bar{\varepsilon}$  the equivalent strain,  $a_0$  the initial cross-sectional radius and  $a$  the current cross-sectional radius. Bridgman model considers the assumptions that axial strain is uniform in the neck cross-section, and that proportional loading conditions take place. Under those assumptions, the radial and tangential strains are equal and double the magnitude of axial strain, therefore, the expression of Equation 4 corresponds also to the equivalent strain. The effect produced by the real distribution of variable strain in the cross-section was studied by Rosa et al. (2003) and Mirone (2004), but for this work, the level of accuracy provided by Bridgman's work has been considered acceptable.

Whilst axial strain varies very little across the neck even beyond necking, axial stress presents a radial variation that has to be taken into account. The material deforms uniformly before necking, and therefore, stress is also uniform. However, when necking starts the stress distribution has to be corrected and the axial stress at any radial position of the cross-section can be obtained using Equation 6:

$$\sigma_{zz} = \frac{\sigma_{zz,av}}{\left(1 + \frac{2\rho}{a}\right) \ln \left(1 + \frac{a}{2\rho}\right)} \left(1 + \ln \left(\frac{a^2 + 2a\rho + r^2}{2a\rho}\right)\right) \quad (6)$$

where  $\sigma_{zz}$  is the axial stress,  $\sigma_{zz,av}$  the average axial stress,  $a$  the neck cross-sectional radius,  $\rho$  the neck radius of curvature and  $r$  the radial distance from the axis of the specimen. If the radial position studied is exactly the centre of the cross-section, the maximum axial stress is obtained. The expression obtained for the equivalent stress in the neck ( $\bar{\sigma}$ ) following Bridgman approach is presented in Equation 7. Equivalent stress only depends on the geometry of the neck ( $a$

250

and  $\rho$ ) and the average axial stress, and it is the parameter that has been used  
 255 to represent the material, along with the equivalent strain of Equation 4.

$$\bar{\sigma} = \frac{\sigma_{zz,av}}{\left(1 + \frac{2\rho}{a}\right) \ln\left(1 + \frac{a}{2\rho}\right)} \quad (7)$$

Round bar specimens (Figure 2) were used to obtain the equivalent stress -  
 equivalent strain behaviour of the materials using the imaging method described  
 above. The same sample geometry was also used to obtain the true stress - true  
 strain of the material using a 6 mm gauge length extensometer and the classical  
 260 processing method (Equations 2 and 3), in order to compare both techniques.

Additional tests were performed on hourglass round specimens (Figure 3), in  
 which strain was measured using both techniques (the imaging and the exten-  
 someter). By doing this, calibration curves for the necking effect correction were  
 produced. These curves associate each measurement of engineering strain and  
 265 stress obtained from the extensometer and load cell to the value of equivalent  
 strain and equivalent stress obtained from the neck geometry at that particular  
 instant of the test. The calibration curves obtained are presented in Figure 8  
 for strains and stresses of stainless steel 304L and copper C110. As expected,  
 the correction curves are lines of slope value  $\sim 1$  before necking, since both  
 270 methods yield the same results. When the neck develops, the slope increases in  
 all the curves due to the localisation of stress and strain that is not captured  
 by the traditional approach. While the increasing slope is easily observed for  
 strains, a backwards or boomerang effect appears in the stress curves. Engineer-  
 ing stress starts decreasing passed the UTS, however, the real equivalent stress  
 275 keeps increasing. The point of highest abscissa of these curves corresponds to  
 the UTS point, after which localisation occurs with either equivalent stress in-  
 creasing further up to failure (stainless steel, Figure 8c), or increasing and then  
 decreasing back to zero, due to the material degradation (copper, Figure 8d).  
 The calibration curves have later been used to account for the necking effects  
 280 in the damage monitoring experiments.

#### 2.4. Experimental procedure for ductile damage modelling

Two different experiments were performed in order to characterise ductile damage, one focused on the quantification of the material degradation when plastic strain is progressively applied, and the other on the ductile fracture strain at various states of stress triaxiality.

To determine the effect of stress triaxiality on fracture plastic strain, tensile tests on round notched bar specimens with different notch geometries were performed (Figure 4 represents a notch of 5 mm minimum diameter and 3 mm notch radius of curvature). The same equipment and room temperature conditions were used for these tests. The imaging technique described for plasticity assessment was also used to monitor the notch throughout the test. This has been especially useful, since it allows the evolution of the stress triaxiality in the centre of the notch to be calculated using the notch's geometrical parameters and Equation 1. The initial triaxiality of the specimen can be obtained from its initial geometry and Equation 1, however, this value varies along the test when the geometry of the notch changes, and the assumption of constant triaxiality would lead to inaccurate material modelling. From the stress triaxiality history, the average value at the centre of the neck was calculated as presented in Equation 8:

$$\eta_{av} = \frac{1}{\bar{\varepsilon}_f} \int_0^{\bar{\varepsilon}_f} \eta(\bar{\varepsilon}) d\bar{\varepsilon} \quad (8)$$

where  $\eta_{av}$  is the average stress triaxiality,  $\eta(\bar{\varepsilon})$  the stress triaxiality history,  $\bar{\varepsilon}$  the equivalent strain and  $\bar{\varepsilon}_f$  the fracture equivalent strain. This average value has later been used for the calibration of models. Generally, fracture strain is obtained by measuring the diameter of the fractured specimen and comparing it with its initial value by means of Equation 4. For validation purposes the final diameter was also measured using a shadowgraph.

To measure the degradation that ductile damage produces in the material, the stiffness reduction method was employed. This technique consists of a tensile test which, instead of running continuously up to fracture, is partially unloaded at different plastic strain stages, and reloaded subsequently. This process is

310 exemplified in Figure 9. From the slope of each of the unloading cycles the damaged or effective elastic modulus ( $\tilde{E}$ ) can be obtained, and from it, an estimation of damage expressed following the continuum damage mechanics definition:

$$D = 1 - \frac{\tilde{E}}{E} \quad (9)$$

where  $D$  is ductile damage,  $E$  the initial elastic modulus and  $\tilde{E}$  the effective or  
 315 deteriorated elastic modulus.

A 250 kN tensile Instron machine was used, in which the successive loading and unloading program was specified so that the test run in a continuous manner in quasi-static conditions ( $\dot{\epsilon} \sim 10^{-3} \text{ s}^{-1}$ ). The hourglass shaped specimens were used for these tests (Figure 3). To measure strain in the gauge region  
 320 for later damage calculation, an extensometer of 6 mm gauge length and a maximum engineering strain measurement capability of 100% was used. In previous studies strain gauges were mainly used for accurate elastic modulus measurement (Bonora (1997); Lemaitre and Dufailly (1987); Alves et al. (2000)). Nevertheless, strain gauges are not able to measure more than 10% plastic strain  
 325 before failing, and therefore, several test interruptions and gauge replacements were required, losing the origin of strain measurement and introducing great inaccuracy in the process. The alternative method used in this research can be performed in a continuous manner reducing considerably the test duration, the specimens preparation complexity and the potential imprecision of strain  
 330 gauge replacement. Although strain gauges could potentially lead to a better approximation of the local strain values due to their smaller size, they cannot be attached accurately to an already necked geometry, which is the region of highest interest. The extensometer used in this research was smaller than those used in previous investigations (Bonora et al. (2011)), and while it still provides  
 335 an averaged strain measurement in the necked region, a closer approximation to the more local measurement of a strain gauge was pursued. Another method to measure strain based on Digital Image Correlation instead of extensometer was initially considered, since it could potentially have led to an even better

approximation of the local strains in the neck, nevertheless, the method was  
discarded due to the higher scatter in the strains measured at such small scale  
(Sancho et al. (2016)).

The measurements obtained from the extensometer was used to create curves  
of stress vs. average strain in the gauge length, as shown in Figure 10, which  
represents an unloading and reloading cycle. The measured values of stress and  
strain were corrected with the curves presented in Figure 8 to account for the  
effects of necking. The unloading measurements were used to calculate elastic  
modulus, as suggested by Lemaitre and Chaboche (1990). The top and bottom  
20% data of each unload cycle were omitted due to the non-linearities in the  
extremes and a fit was made to the linear region to determine the effective  
elastic modulus. By comparing this effective modulus with the initial value,  
ductile damage can be calculated using Equation 9.

As previously mentioned, the effects of cross-section reduction and necking  
on stress and strain have been taken into account. This has been done by  
interpolating the raw data obtained from the extensometer and load cell with  
the calibration curves of Figure 8 obtained from the plasticity tests, which relate  
measured engineering values with equivalent corrected values.

### *2.5. Plasticity and damage models*

The experimental results obtained were used to calibrate the quasi-static  
and room temperature part of Johnson-Cook plastic and damage model (John-  
son and Cook (1983, 1985)) with the new damage accumulation law proposed.  
Due to the quasi-static strain-rate and constant temperature of the experiments  
considered in this work, the models reduce to a power-law expression for plastic  
behaviour and an exponential expression for ductile damage, each of them with  
3 parameters to be calibrated. Alternative models such as a classical power law  
for the plastic behaviour representation, and an exponential Rice and Tracey  
like model for the ductile fracture (each with only 2 parameters) could also  
be used. However, in this case the simplified versions of Johnson-Cook model  
were selected, as the work has been extended to different strain-rates and tem-

peratures (Sancho et al. (2018)). The hardening law is defined by Equation 10,  
 370 where the first term represents the isotropic plastic strain hardening, the second  
 term the strain-rate hardening and the third term the temperature softening:

$$\bar{\sigma} = [A + B\bar{\varepsilon}_{pl}^n] \left[ 1 + C \ln \left( \frac{\dot{\bar{\varepsilon}}_{pl}}{\dot{\bar{\varepsilon}}_0} \right) \right] \left[ 1 - \left( \frac{T - T_{ref}}{T_m - T_{ref}} \right)^m \right] \quad (10)$$

where  $\bar{\sigma}$  represents the von Mises equivalent stress,  $\bar{\varepsilon}_{pl}$  the equivalent plastic  
 strain,  $\dot{\bar{\varepsilon}}_{pl}$  the equivalent plastic strain-rate,  $\dot{\bar{\varepsilon}}_0$  the reference strain-rate,  $T$  the  
 temperature,  $T_m$  the melting temperature,  $T_{ref}$  the reference temperature and  
 375  $A, B, n, C$  and  $m$  the model parameters. Some authors use the room tempera-  
 ture as  $T_{ref}$  and others use the lowest temperature studied, hence care must be  
 taken to keep consistent its definition.

To represent the ductile fracture point of the hardening curve, Johnson and  
 Cook (1985) model has been used. The expression of fracture equivalent strain  
 380 is similar to that of the flow rule with a term representing the effect of stress  
 triaxiality on fracture, a second term the effect of strain-rate and a third term  
 the effect of temperature:

$$\bar{\varepsilon}_{pl,f} = [D_1 + D_2 \exp(D_3 \eta)] \left[ 1 + D_4 \ln \left( \frac{\dot{\bar{\varepsilon}}_{pl}}{\dot{\bar{\varepsilon}}_0} \right) \right] \left[ 1 + D_5 \left( \frac{T - T_{ref}}{T_m - T_{ref}} \right) \right] \quad (11)$$

where  $\bar{\varepsilon}_{pl,f}$  represents the fracture equivalent plastic strain,  $\eta$  the stress triaxial-  
 ity,  $D_1, D_2, D_3, D_4$  and  $D_5$  the damage model parameters, and the strain-rates  
 385 and temperatures have the same meaning as those in Equation 10.

In the original Johnson-Cook damage model a basic linear damage accumu-  
 lation of the following form was presented:

$$D = \int_0^{\bar{\varepsilon}_{pl}} \frac{d\bar{\varepsilon}_{pl}}{\bar{\varepsilon}_{pl,f}} \quad (12)$$

Fracture was considered to occur when the accumulated damage reached a crit-  
 ical value of  $D = 1$ , i.e. when  $\bar{\varepsilon}_{pl} = \bar{\varepsilon}_{pl,f}$ .

390 After analysing the damage accumulation experimental results it has been  
 concluded that this linear approximation does not represent the real behaviour  
 of the studied materials and a different accumulation law has been proposed. It  
 is based on Cortese et al. (2016) damage accumulation model that was derived

for ductile damage by making a direct parallelism with the high cycle fatigue  
 395 damage law proposed by Palmgren (1924), but including also a non-linear power  
 law for damage accumulation. For this work, Cortese model has been modified  
 so that some concepts of continuum damage mechanics, such as critical damage  
 ( $D_{cr}$ ) and threshold plastic strain ( $\varepsilon_{th}$ ), also appear in the equation:

$$D = D_{cr} \left( \frac{\bar{\varepsilon}_{pl} - \varepsilon_{th}}{\bar{\varepsilon}_{pl,f} - \varepsilon_{th}} \right)^{\left( \frac{m}{\bar{\varepsilon}_{pl,f}} \right)^q} \quad (13)$$

It includes an exponent which depends on the stress state through the fracture  
 400 equivalent plastic strain ( $\bar{\varepsilon}_{pl,f}$ ) and on the material by means of the model  
 parameters  $m$  and  $q$ ,  $\bar{\varepsilon}_{pl}$  represents the accumulated equivalent plastic strain,  
 $\varepsilon_{th}$  the threshold plastic strain and  $D_{cr}$  the critical damage. The effect of  
 varying the model parameters  $m$  and  $q$  is represented in Figure 11.

Consequently, the approach followed for damage and fracture modelling is  
 405 mainly empirical, based on Johnson-Cook fracture model (Equation 11) and  
 introducing a new non-linear damage accumulation law (Equation 13). This  
 accumulation model is also of empirical nature, however, due the shape of the  
 equation, some of its parameters represent actual physical behaviours of the  
 material (threshold plastic strain and critical damage), the same as parame-  
 410 ter  $A$  in Johnson-Cook plastic model (Equation 10) represents the yield stress.  
 Therefore, one of the benefits of the proposed model is that it provides some ad-  
 ditional information about the material damage whilst maintaining its empirical  
 nature.

Some investigations have been conducted on the effect of stress state on the  
 415 ductile damage accumulation law (Papasidero et al. (2015); Fincato and Tsut-  
 sumi (2019)). The model proposed here uses Johnson-Cook fracture criterion to  
 calculate strain to fracture, which considers the effect of stress triaxiality, how-  
 ever, does not include the effect of Lode angle. Therefore, the model may not  
 be able to represent the phenomena observed for stress states which are consid-  
 420 erably different to the range analysed in this work. Nonetheless, this research  
 is focused on the development of the experimental and modelling techniques  
 required to measure and represent non-linear damage accumulation behaviours,



and the effect of a wider range of stress states could be analysed and included in the model in the future.

### 425 **3. Experimental results**

#### *3.1. Uniaxial tests results*

The equivalent plastic stress and strain behaviour of the material was determined by applying the neck monitoring technique described in Section 2.3. The results presented here correspond to Test No. 1 (see Table 1). From the images  
430 post-processing the history of specimen minimum radius ( $a$ ) and neck radius of curvature ( $\rho$ ) can be obtained. The evolution of these geometrical parameters is represented in Figure 12a for stainless steel 304L and in Figure 12b for copper C110.

As observed, the cross-sectional radius reduces approximately uniformly with  
435 applied strain for both materials. The other parameter represented is the neck radius of curvature, which presents an infinite value before necking starts due to the cylindrical geometry of the specimen. At the onset of necking this value decreases more or less quickly depending on how rapid the neck development is. Apart from the higher ductility and cross-section reduction of copper compared  
440 to stainless steel, the onset of necking location of both materials can be compared in Figure 12 and Table 2. While stainless steel presents a clear region of uniform plastic deformation up to  $\sim 35\%$  strain, copper necks almost from the beginning of the plastic region, allowing only  $\sim 4\%$  of strain before the instability starts. The values of final specimen cross-section measured with the shadowgraph are  
445 also included in Table 2, and it can be appreciated that the values are very similar to those obtained with the imaging method, validating the latter.

Making use of the geometrical parameters presented above, the equivalent stress - equivalent strain curve can be obtained. Figure 13a shows the results for stainless steel 304L, where results of two additional conventional tensile tests  
450 are also presented. For these last two tests, strain has been measured using an extensometer and Equations 2-3 have been used to post-process the results. As

expected, the results of both methods coincide before the specimen necks and the traditional approach cannot be used during necking, where the real stress and strain are no longer uniform. The same results are presented for copper C110 in Figure 13b, where a conventional tensile test has also been plotted. The outcome is the same as that of the stainless steel, being in this case even more pronounced due to the early necking of copper. The plastic strain - stress obtained for this material using Equations 2-3 is inappropriate, since it necks almost from the beginning of the test. The fracture mode is also different in both materials, while the stainless steel fails suddenly when it reaches its failure strain, the copper fails progressively with high ductility, hence the gradual stress decrease observed in Figure 13b.

### 3.2. Triaxiality effects results

The effect of stress triaxiality on ductility has been studied by testing round notched bar specimens with different notch geometries. The results correspond to Tests No. 3 - 8 of Table 1, but results of Test No. 1 are also included here, since they represent the extreme case of a notch with an infinite notch radius.

The same imaging methodology previously presented for smooth specimens has been employed here for the analysis of the notched specimens. Tracking the curvature of the neck, present from the beginning in these specimens, the evolution of the geometrical parameters  $a$  (cross-sectional radius) and  $\rho$  (neck radius of curvature) has been obtained. The neck reduction as a function of equivalent plastic strain is governed by Equation 4, and results in an exponentially decreasing curve that only differs for each specimen geometry on the fracture strain point, which becomes smaller for higher triaxialities.

The evolution of the other parameter, the neck radius of curvature, is presented in Figure 14a for the stainless steel and Figure 14b for the copper. Both materials follow similar trends, presenting small curvature variation the sharper notches (almost no variation for stainless steel) and considerable curvature decrease the smoother geometries. The curvature at fracture is slightly smaller for the copper compared to the stainless steel. For each of the materials all the

specimens fail at similar values of curvature, with a weak ascending trend for decreasing triaxialities.

The equivalent stress - equivalent strain curves of all the tests conducted  
485 and obtained using the imaging method and Bridgman theory are presented  
in Figure 15a for stainless steel 304L and Figure 15b for copper C110. In  
Figures 14-15 the yellow curves correspond to the round bar tensile test (Test  
No. 1), and have been previously presented in Figure 12 for radius of neck cross-  
section and radius of curvature and in Figure 13 for stress-strain. The rest of  
490 curves depict the different notch geometries tested with the expected trend of  
decreasing fracture equivalent strain for increasing triaxiality already observed  
in the  $a$  and  $\rho$  plots. In Figure 15 it can be observed that all the curves have  
the same yield stress and hardening rate, proving that stress triaxiality does not  
affect the early plastic region, and only affects the final fracture conditions. A  
495 difference is observed in the plastic strain for which copper starts undergoing  
severe degradation, which follows the same trend as fracture strain, appearing  
earlier the higher the stress triaxiality.

If the maximum axial stress is analysed (Figure 16), it can be observed that  
most stainless steel specimens fracture at a similar maximum axial stress value,  
500 independently of the stress triaxiality. In the case of copper, it is the initiation of  
the severe degradation process which occurs at the maximum axial stress of the  
material, while instead of failing in a brittle manner, a subsequent degradation  
occurs before final fracture. The value of maximum axial stress in the centre  
of the neck is therefore, in both cases, the physical limit that the material can  
505 withstand in tensile conditions and is not affected by stress triaxiality.

From a damage modelling point of view, it is well known that fracture equiv-  
alent strain decreases exponentially with stress triaxiality (Johnson and Cook  
(1983); Bai and Wierzbicki (2008)); however, the results of this investigation  
are presented in Figure 17, where the stress triaxiality history of each specimen  
510 obtained with the imaging method has also been plotted. The crosses represent  
the fracture strain conditions against the average stress triaxiality throughout  
the test, which in some cases varies considerably. The curves for stainless steel

are smooth up to fracture while the copper results present some fluctuation at the end. This occurs because the neck developed by the steel presents a smooth curvature during the whole deformation process; the neck of the copper specimens, however, is much sharper at the last stages of plastic deformation, making it very difficult to determine the curvature. If the yellow lines obtained from the smooth samples are analysed, it can be observed that although that geometry is generally used to represent uniaxial conditions, these are only met during the uniaxial deformation region, and a considerable increase of stress triaxiality takes place afterwards.

The selection of a stress triaxiality value for the test supposes a difficulty for fracture models calibration. In some works the initial or final stress triaxialities have been used, nonetheless, this implies a large error for some geometries and, since in this case the triaxiality history is known, the use of the average triaxiality given by Equation 8 has been considered the most adequate alternative. This approach was already used by Wierzbicki et al. (2005), making use in that case of calibrated finite elements simulations to estimate the stress triaxiality evolution, and thus the average triaxiality.

Detailed results with the information presented in Figure 17 are also available in Tables 3 and 4 for stainless steel and copper respectively. For each specimen the theoretical value of stress triaxiality predicted by Equation 1 is firstly shown. Since this value changes through the experiment, the values of initial, final and average triaxiality calculated with the imaging method are presented. Finally, the fracture equivalent strain obtained also with the imaging technique is shown. The same value obtained from the post-test specimen diameter measurement using a Starret HE400 optical comparator and Equation 4 is also displayed for comparison and validation of the imaging methodology.

### 3.3. Damage accumulation results

The outcome of the last experiment conducted is the evolution of ductile damage accumulated as the equivalent plastic strain progresses for the uniaxial loading case. These results complement those of the fracture conditions by

characterising the degradation process that occurs between damage initiation and final fracture. They correspond to Test No. 2 of Table 1, for which the  
545 hourglass shaped specimen geometry has been used.

The effective elastic modulus measurement method with a small extensometer, as previously described, has been used to obtain the results presented in Figure 18. The results of four stainless steel 304L specimens are shown in Figure 18a and those of three copper C110 specimens in Figure 18b. As observed,  
550 the repeatability of results is quite good in the case of stainless steel, with a maximum standard deviation of all the elastic modulus measurements taken at a particular plastic strain of 3.04 GPa, and an average standard deviation of the measurements taken at different plastic strains of 0.86 GPa. The results display a similar shape to that presented in Bonora et al. (2011) for another steel. For  
555 copper, the results show a bigger scatter (maximum standard deviation of 3.31 GPa and average standard deviation of 1.53 GPa in the measurement of elastic modulus), and the trend observed is clearly different to that of the stainless steel. A very similar material was also studied in Lemaitre and Dufailly (1987) and Bonora et al. (2011), and the behaviour reported is in close agreement with  
560 that obtained in the present research.

The stainless steel shows a prominent initial increase of damage that progressively stabilizes to a quite low critical damage value (the same behaviour has been observed in microstructural measurements, as will be presented in the following section). For copper, damage does not appear at the early stages of  
565 deformation and then increases fairly linearly. This exemplifies the different damage accumulation behaviours that diverse materials can present and the requirement of a non-linear damage growth model that allows to modify the shape of the accumulation. The appearance of ductile damage at initial stages of plasticity is generally associated to the nucleation of voids due to the fracture  
570 of hard particles within the material, therefore, the results suggest that these defects appear in stainless steel 304L as soon as plastic deformation is applied, however, some plastic strain can be applied to copper C110 before damaging the material.

For copper, it can be observed in Figure 18b that there are no damage  
575 measurements beyond an equivalent plastic strain of around  $\bar{\epsilon}_{pl} = 1$ . This is  
because from that point onwards the material is so degraded, as observed in  
Figure 13b, that unloading and reloading cycles cannot be performed without  
fracturing the specimen; therefore, damage has only been measured up to the  
severe degradation start point and not until final fracture conditions.

#### 580 3.4. Microstructural analysis

A second experimental method has been applied to measure damage and  
validate the experimental results obtained with the elastic modulus reduction  
technique. This has consisted in the analysis of internal porosity in the mini-  
mum cross-section of specimens with the same geometry of those used for the  
585 elastic modulus analysis (HGR6, see Figure 3) strained to different levels of  
plastic deformation. It has been achieved through a metallographic study of  
six stainless steel 304L specimens that have been plastically deformed to the  
plastic strain levels specified in Table 5. The samples were deformed in the  
same quasi-static and room temperature conditions used to obtain the elastic  
590 modulus results. After being strained, the central region of the specimens was  
cut using a Struers Accutom-5 high precision cut-off machine, mounted and pol-  
ished using an ATA Saphir 520 semi-automatic polisher down to 0.1  $\mu\text{m}$  using  
OP-S colloidal silica suspension as the final stage. A Zeiss compound optical  
microscope has been used to take pictures of their microstructure. The main  
595 objective of the investigation has been to quantify the porosity or void density  
in the micrographs, defined as percentage of the total area occupied by voids. A  
thresholding algorithm was programmed in Matlab to batch process the images  
from the microscope, and measurements of porosity have been taken in different  
locations of the specimen's cross-section.

600 Dark-field microscopy is recommended to detect voids, since the images have  
provided an improved contrast, making it easier to differentiate the defects from  
the material during the thresholding process. A square region of size 156  $\mu\text{m}$  x  
156  $\mu\text{m}$  in the centre of the picture was analysed in order to avoid brightness and

contrast peaks from the corners. The threshold value used in the images post-  
605 processing was calibrated by analysing the zoomed-in image of individual voids,  
both in the original and the thresholded picture. Their size has been compared  
and the threshold modified until the size of both voids has been reasonably  
similar, as shown in Figure 19.

The post-process of micrographs has provided the porosity contour maps in  
610 the neck cross-section of Figure 20, where the first sample corresponds to the  
undamaged raw material, and the other six to the strained samples presented in  
Table 5. In these plots, the final diameters of all samples are proportional to the  
real values and the colour scale is also the same in all of them except in the last  
one, which presents much higher void density than the rest. From the analysis  
615 it was observed that the porosity distribution in the raw material is almost per-  
fectly uniform, of very low value, and can be attributed to the inherent defects  
present in the material. Porosity is also fairly uniform in the cross-section of the  
following four specimens, the lighter blue in the porosity contours represents the  
higher void density, nevertheless, porosity increases significantly for specimen  
620 # 34 ( $\bar{\epsilon}_{pl} = 0.11$ ), and almost stabilises for the following three specimens, vali-  
dating the behaviour observed from the elastic modulus measurements. Finally,  
the last two specimens were deformed beyond the necking limit, presenting a  
much higher porosity in general, and a higher concentration in the centre of  
the specimen than near its surface, especially specimen # 6, which had already  
625 fractured.

The porosity results of the contour plots have been plotted together in Fig-  
ure 21, where the evolution of void density along the plastic range can be anal-  
ysed. For each sample, the average of all the measurements taken in the cross-  
section has been represented. As observed, the trend is the same described  
630 above, with a significant increase of porosity in the first stages of plastic defor-  
mation, and a subsequent stabilisation and slight increase near fracture. The  
mean values of ductile damage from Figure 18a obtained by measuring changes  
in elastic modulus have also been included. Although the scale is different for  
the void density and the macroscopic damage, it can be observed that both mea-

635 surements follow the same trend. These results prove that the ductile damage  
 measured from elastic modulus degradation is driven by voids nucleation, growth  
 and coalescence. However, they also suggest that the reduction of mechanical  
 properties does not take exactly the same value as the area reduction that these  
 voids produce in the cross-section, as is generally asserted by continuum damage  
 640 mechanics formulation. Therefore, additional degradation processes linked to  
 the density of voids, but not entirely to the area that they occupy, have to be  
 responsible of the reduction of material properties and will be studied in the  
 future. From the degradation of elastic modulus, which is an actual macroscopic  
 quantifiable property, a value of ductile damage can be obtained and used in  
 645 material models to perform useful engineering calculations. The microscopic  
 analysis carried out in this section serves as validation of the trends followed by  
 the ductile damage parameter, but it is considered that the elastic modulus ap-  
 proach can provide a more real representation of the real macroscopic operation  
 of components.

#### 650 **4. Model calibration**

The equivalent stress - equivalent strain results obtained through the imaging  
 technique applied to tensile specimens (Figure 13) have been used to calibrate  
 the isotropic hardening part of Johnson-Cook plastic model (Equation 10), to  
 represent the plastic behaviour of the studied materials. The experimental stress  
 655 - strain results and the fitted plastic model curve are presented in Figure 22 and  
 the model parameters obtained summarised in Table 6. The fracture equivalent  
 plastic strain - stress triaxiality results of Figure 17 have been used to calibrate  
 the quasi-static and room temperature part of Johnson-Cook fracture model  
 (Equation 11). The experimentally obtained data points as well as the model  
 660 fitted curve and the model parameters for stainless steel 304L and copper C110  
 are represented in Figure 23. In a similar manner, the results obtained in the  
 damage accumulation experiments (Figure 18) have been fitted to the proposed  
 damage accumulation law of Equation 13.



In this case damage evolves with equivalent plastic strain ( $\bar{\varepsilon}_{pl}$ ) and with fracture equivalent plastic strain, and hence with stress triaxiality ( $\bar{\varepsilon}_{pl,f} = f(\eta)$ ), according to the non-linear model. The variables to be calibrated based on the experimental evidence are  $D_{cr}$ ,  $\varepsilon_{th}$ ,  $q$  and  $m$ , as well as the variables of Johnson-Cook model embedded in the fracture equivalent plastic strain expression ( $D_1$ ,  $D_2$  and  $D_3$ ). The experimental damage points can be fitted to a three dimensional surface against equivalent plastic strain and stress triaxiality axes ( $D = f(\bar{\varepsilon}_{pl}, \eta)$ ), as presented in Figure 24. The crosses represent the damage - equivalent plastic strain values already plotted on Figure 18, while the triaxiality data has been obtained from the neck monitoring technique, following a similar procedure to that used to obtain the engineering to equivalent strain corrections.

The parameters  $D_1$ ,  $D_2$  and  $D_3$  of Johnson-Cook model have been calibrated by least squares fitting of the fracture experimental data (fracture equivalent plastic strain vs. average stress triaxiality throughout the experiment) to Equation 11.  $D_{cr}$ ,  $\varepsilon_{th}$ ,  $m$  and  $q$  of the accumulation law have also been obtained by least squares fitting of the damage accumulation data to Equation 13 using the previously fitted Equation 11 expression as input. The calibrated variables obtained for both materials have been summarised in Table 7.

## 5. Conclusions

The proposed advanced methodology was successfully employed to obtain the post-necking equivalent stress - strain measurements of 304L stainless steel and C110 copper. The accuracy of the approach was validated by comparison with traditional tensile results in the uniform deformation region and with post-fracture geometrical measurements. The technique was applied using affordable equipment, making it accessible and of high value for industrial applications.

The measurements of stress triaxiality demonstrated that this parameter changes considerably throughout the tests, highlighting the fact that a constant value cannot be assumed. In this research, the use of the average measured

value is proposed as a way of accounting for the variation of stress triaxiality when calibrating the fracture model.

695 A highly non-linear ductile damage behaviour was measured on 304L stainless steel and C110 copper. Additionally, these materials showed significant differences in their damage accumulation behaviour. The above findings demonstrated the need of a complex and versatile ductile damage model that is able to represent non-linearity and different damage accumulation shapes.

700 The damage accumulation behaviour was validated by a microstructural analysis, which provided estimations of porosity or void density along the plastic deformation process. The trends followed by the increase in void density and the degradation of elastic modulus are coincident. However, the magnitude of modulus decrease is much higher than that of porosity increase, suggesting that  
705 void density does determine the shape of damage accumulation, but additional degradation phenomena proportional to void density occur within the material and require further investigation.

The proposed ductile damage model includes non-linear behaviour and has been successfully calibrated using the experimental data gathered. The capability of representing accurately damage accumulation makes the model an effective  
710 solution in, for example, the automotive and transport industry, where accurate damage prediction in collisions is important for safe and efficient component design. Also, the approach is of much value to components manufactured using metal forming where very high plastic deformation can occur and there is a need  
715 to avoid failure by ductile damage.

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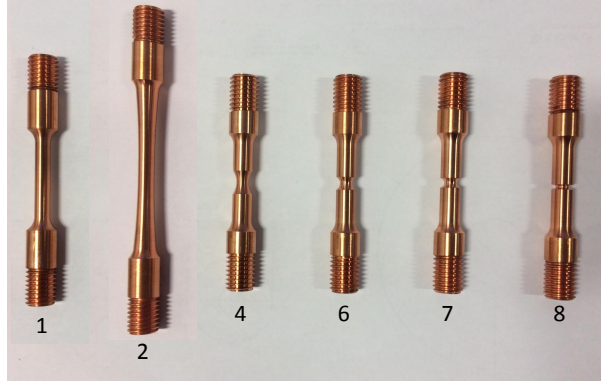


Figure 1: Geometry of copper specimens tested (for stainless steel, specimens 3 and 5, as defined in Table 1, have been used instead of specimen 4).

Test number	Specimen characteristics	Notch acuity, $a/\rho$ [-]	Curvature, $\rho$ [mm]	Diameter, $2a$ [mm]	Theoretical triaxiality, $\eta_0$ [-]
1	Smooth bar	0	$\infty$	6	0.333
2	Hourglass shaped bar	0	$\infty$	6	0.333
3	Notched bar	0.25	8	4	0.557
4		0.313	8	5	0.605
5		0.417	6	5	0.682
6		0.833	3	5	0.940
7		1.25	2	5	1.144
8		2.5	1	5	1.586

Table 1: Characteristics of the analysed specimens.

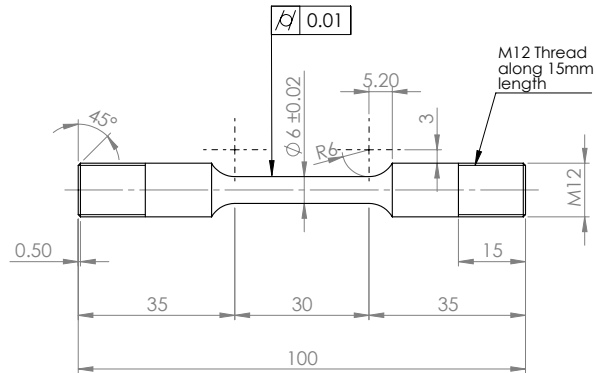


Figure 2: RB6 specimen geometry.



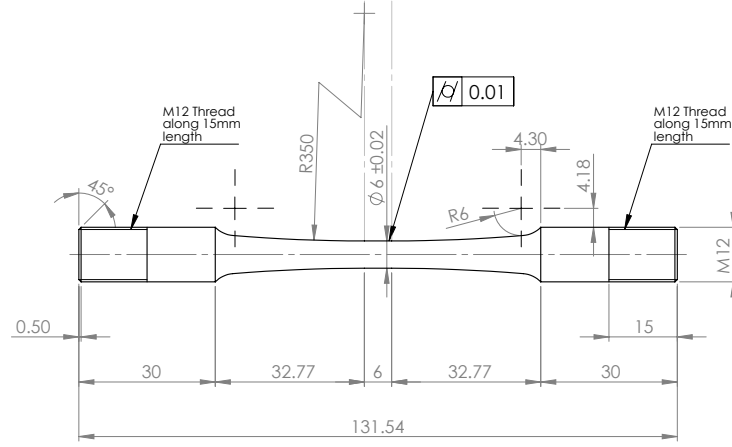


Figure 3: HGR6 specimen geometry.

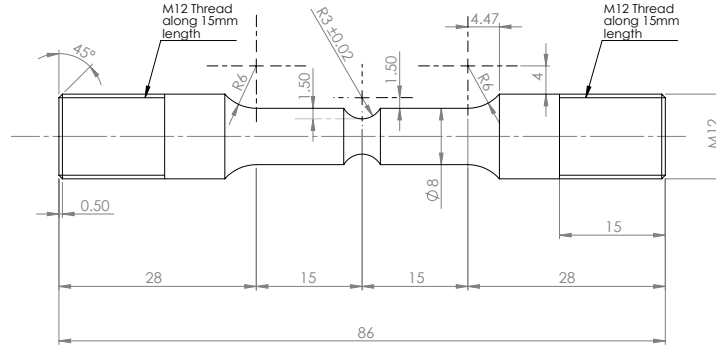


Figure 4: RNB6 specimen geometry ( $a = 5$  mm,  $\rho = 3$  mm).

Material	Geom.	Spec.	Imaging			Shadowgraph
			$\epsilon_{onset}$ [-]	$\rho_f$ [mm]	$a_f$ [mm]	$a_f$ [mm]
s/s	RB6	#14	0.35	2.62	1.50	1.49
304L		#16	0.38	2.58	1.51	1.53
copper	RB6	#1	0.05	0.94	1.29	1.27
C110		#2	0.05	0.88	1.22	1.22

Table 2: Neck onset strain ( $\epsilon_{onset}$ ), final neck radius of curvature ( $\rho_f$ ) and final cross-sectional radius ( $a_f$ ) measured with imaging method; final cross-sectional radius ( $a_f$ ) measured with shadowgraph.

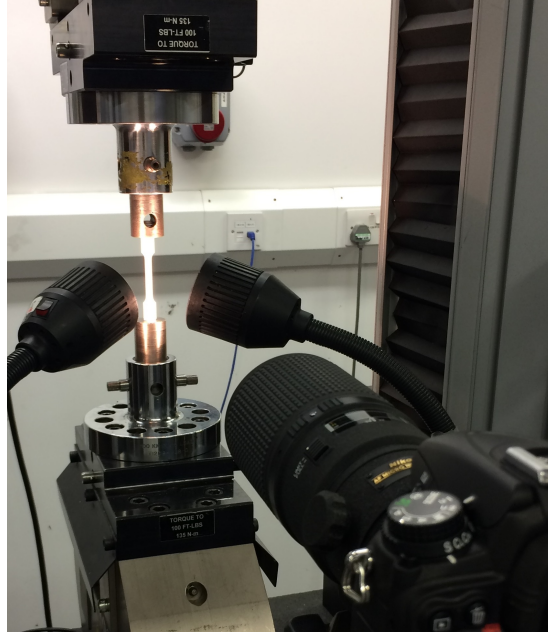


Figure 5: Experimental rig used for the specimen's neck geometry monitoring.

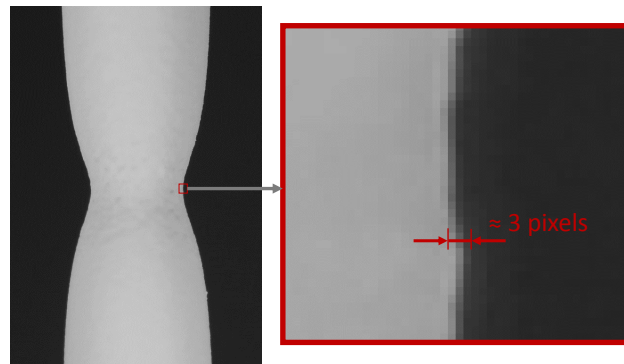


Figure 6: Representation of the accuracy of the neck monitoring technique, thickness of the grey transition between dark and light pixels of the image.

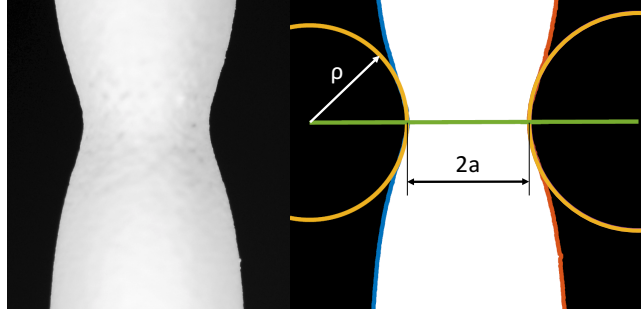
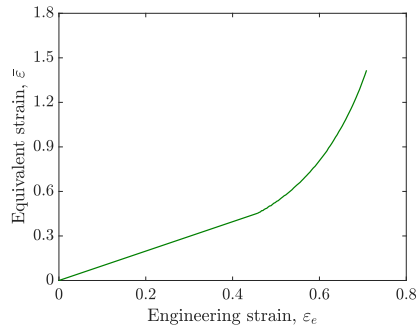
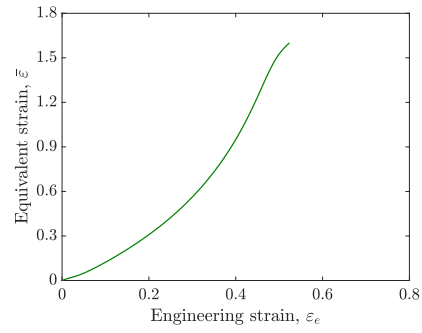


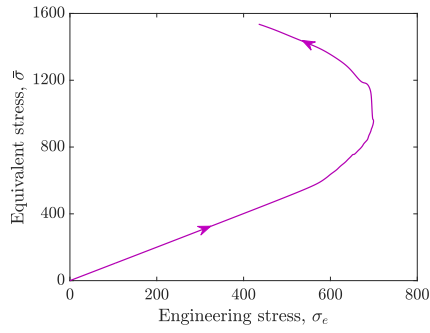
Figure 7: Representation of the necked specimen obtained in the condition of  $\varepsilon_T = 1.3$  for a stainless steel 304L specimen. The neck shape, minimum diameter ( $2a$ ) and osculating circles are shown.



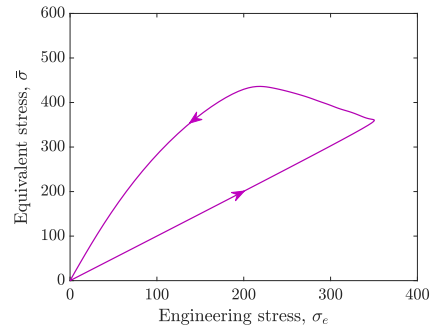
(a) Strain correction for stainless steel 304L



(b) Strain correction for copper C110



(c) Stress correction for stainless steel 304L



(d) Stress correction for copper C110

Figure 8: Necking effect correction curves: strain correction ( $\bar{\varepsilon} - \varepsilon_e$ ) and stress correction ( $\bar{\sigma} - \sigma_e$ ).

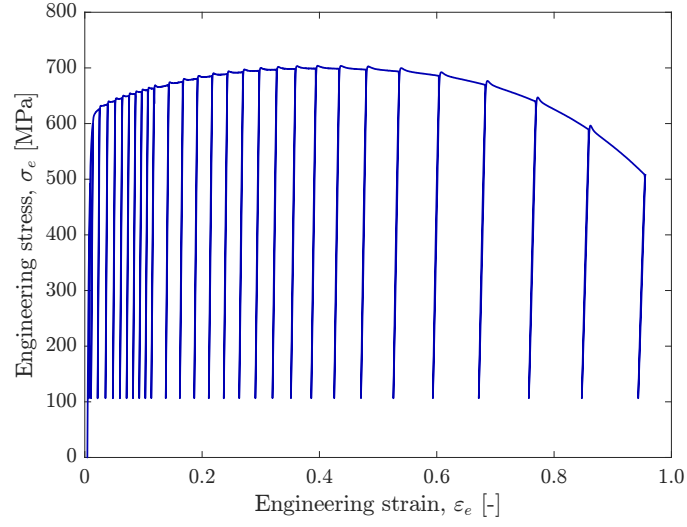


Figure 9: Engineering stress - engineering strain history of the elastic modulus monitoring test.

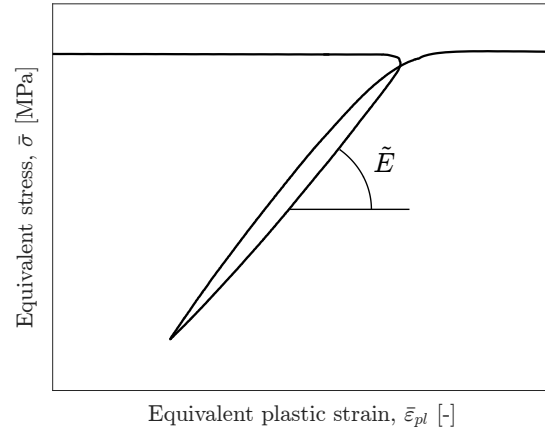


Figure 10: Equivalent stress - equivalent plastic strain ( $\bar{\sigma}$  -  $\bar{\varepsilon}_{pl}$ ) of one unloading-reloading cycle.

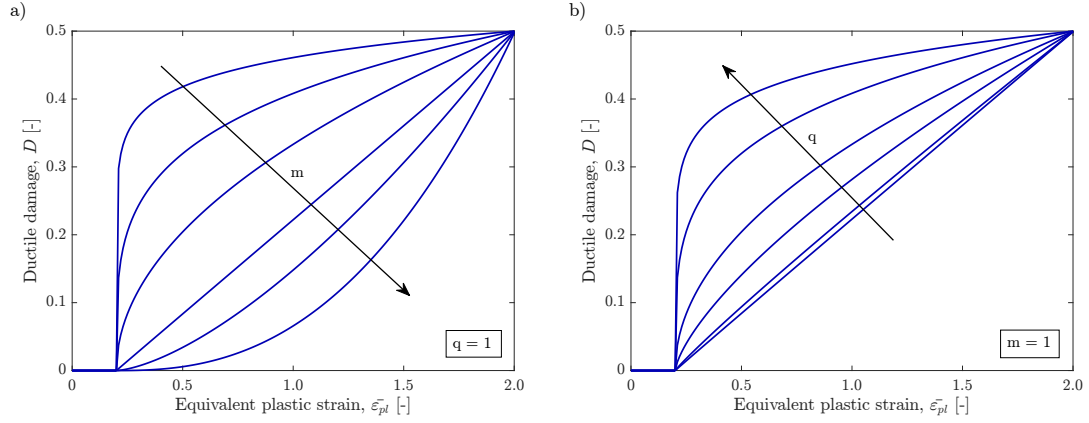


Figure 11: Effect of  $m$  and  $q$  parameters on the damage accumulation model: (a)  $q = 1$  and variable  $m$ ; (b)  $m = 1$  and variable  $q$ . In both cases  $\epsilon_{th} = 0.2$  and  $D_{cr} = 0.5$ .

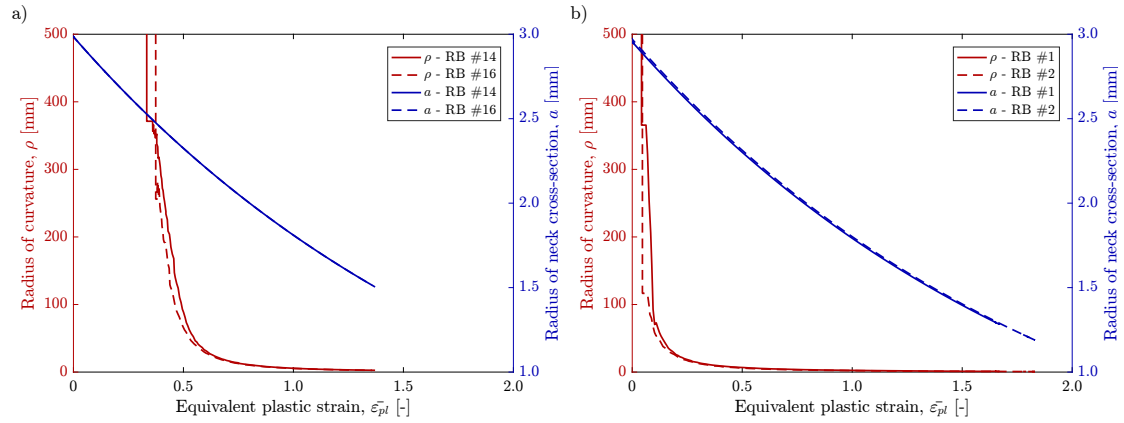


Figure 12: Evolution of the neck geometry (minimum cross-sectional radius ( $a$ ) and radius of curvature of the osculating circle ( $\rho$ )) vs. equivalent plastic strain: (a) stainless steel 304L; (b) copper C110.

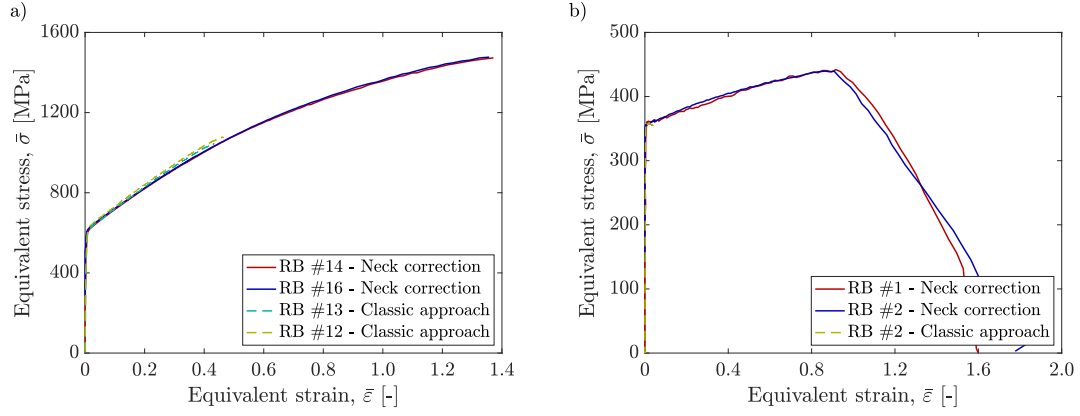


Figure 13: Equivalent stress - equivalent strain: (a) stainless steel 304L; (b) copper C110.

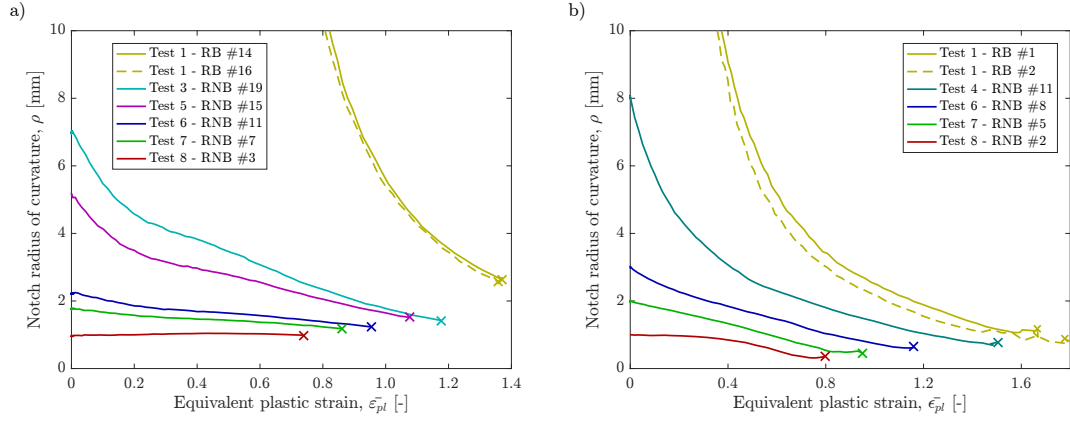


Figure 14: Evolution of the radius of curvature of the notch with applied equivalent plastic strain for the different RNB tensile tests: (a) stainless steel 304L; (b) copper C110.

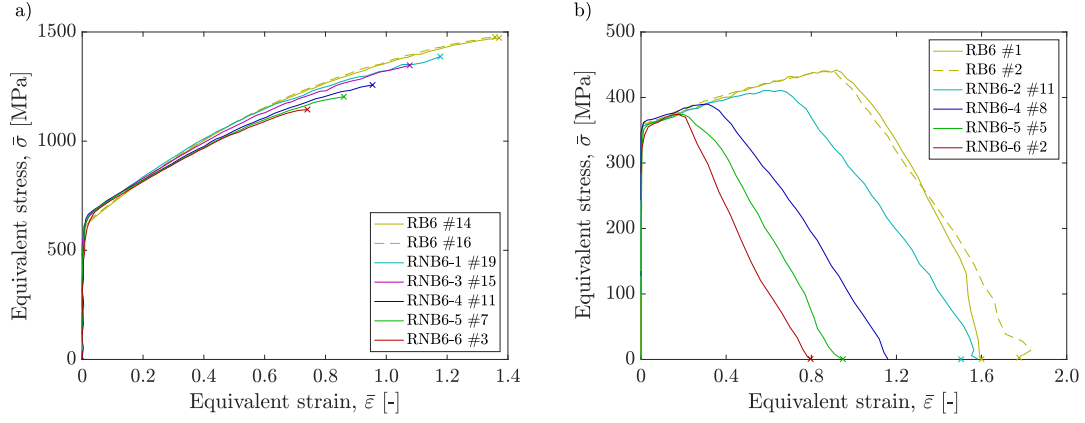


Figure 15: Equivalent stress - equivalent strain of the RNB tensile tests: (a) stainless steel 304L; (b) copper C110.

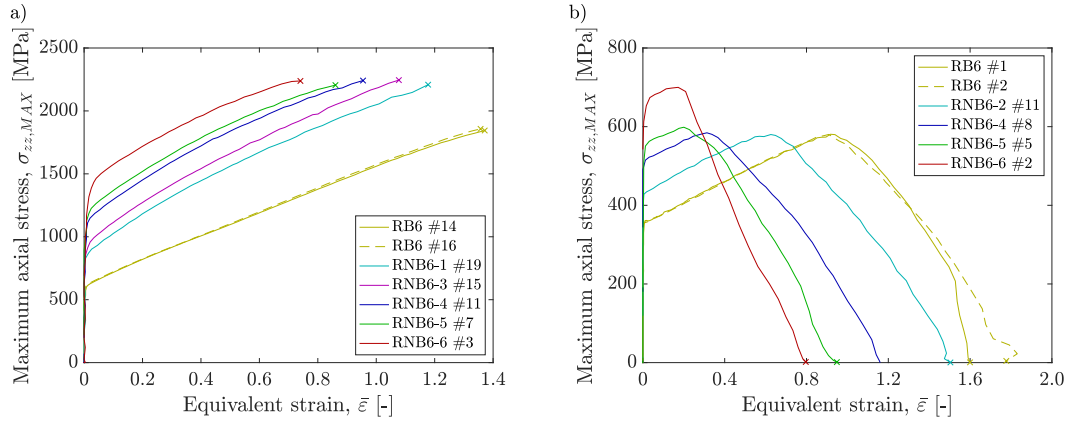


Figure 16: Maximum axial stress - equivalent strain of the RNB tensile tests: (a) stainless steel 304L; (b) copper C110.

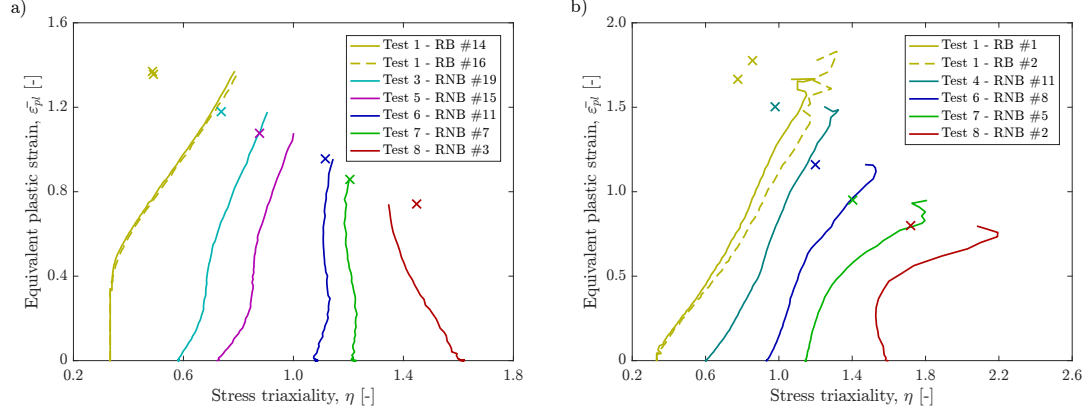


Figure 17: Variation of stress triaxiality with applied equivalent plastic strain for the different RNB tensile tests, the crosses represent the fracture equivalent plastic strain vs. average triaxiality ( $\bar{\epsilon}_{pl,f}, \eta_{av}$ ): (a) stainless steel 304L; (b) copper C110.

Test #	Notch		Theor.	Imaging					Shadowgr.
	acuity,	Spec.		$\eta_0$	$\eta_0$	$\eta_f$	$\eta_{av}$	$\bar{\epsilon}_f$	
	$a/\rho$ [-]		$\eta_0$	$\eta_0$	$\eta_f$	$\eta_{av}$	$\bar{\epsilon}_f$	$\bar{\epsilon}_f$	
1	0	#14	0.333	0.333	0.787	0.491	1.37	1.39	
		#16	0.333	0.333	0.796	0.494	1.36	1.34	
3	0.25	#19	0.546	0.582	0.905	0.742	1.18	1.18	
5	0.417	#15	0.660	0.726	1.001	0.880	1.08	1.04	
6	0.833	#11	0.892	1.084	1.144	1.117	0.96	0.96	
7	1.25	#7	1.110	1.225	1.203	1.207	0.86	0.88	
8	2.5	#3	1.540	1.614	1.346	1.447	0.74	0.75	

Table 3: Stress triaxiality results of stainless steel 304L. Theoretical initial stress triaxiality ( $\eta_0$ ) obtained using Eq. 1; initial ( $\eta_0$ ), final ( $\eta_f$ ) and average ( $\eta_{av}$ ) stress triaxiality throughout the test obtained using the proposed imaging method; and fracture equivalent strain ( $\bar{\epsilon}_f$ ) obtained using the imaging technique and measuring the final diameter.



Test #	Notch		Theor.	Imaging				Shadowgr.
	acuity,	Spec.		$\eta_0$	$\eta_f$	$\eta_{av}$	$\bar{\varepsilon}_f$	
	$a/\rho$ [-]		$\eta_0$	$\eta_0$	$\eta_f$	$\eta_{av}$	$\bar{\varepsilon}_f$	$\bar{\varepsilon}_f$
1	0	#1	0.333	0.333	1.397	0.823	1.60	1.69
		#2	0.333	0.333	1.316	0.867	1.78	1.78
4	0.313	#11	0.606	0.602	1.324	0.981	1.50	1.49
6	0.833	#8	0.958	0.936	1.530	1.198	1.16	1.19
7	1.25	#5	1.173	1.147	1.808	1.403	0.95	0.98
8	2.5	#2	1.532	1.586	2.081	1.721	0.80	0.77

Table 4: Stress triaxiality results of copper C110. Theoretical initial stress triaxiality ( $\eta_0$ ) obtained using Eq. 1; initial ( $\eta_0$ ), final ( $\eta_f$ ) and average ( $\eta_{av}$ ) stress triaxiality throughout the test obtained using the proposed imaging method; and fracture equivalent strain ( $\bar{\varepsilon}_f$ ) obtained using the imaging technique and measuring the final diameter.

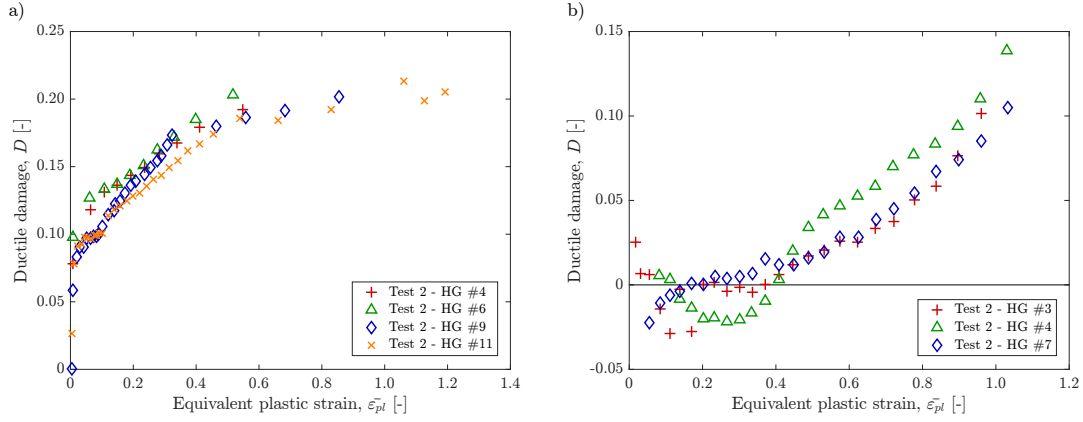


Figure 18: Ductile damage - equivalent plastic strain experimental results: (a) stainless steel 304L; (b) copper C110.

Specimen	#34	#33	#32	#36	#31	#6
Equivalent plastic strain ( $\bar{\varepsilon}_{pl}$ )	0.110	0.214	0.384	0.673	0.993	1.292

Table 5: Specimens studied in the metallographic damage analysis and equivalent plastic strains.

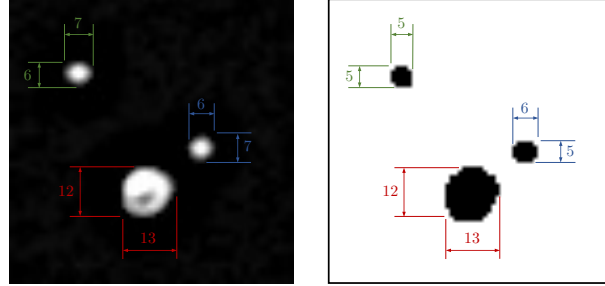


Figure 19: Size comparison of individual voids in the original and thresholded pictures for the calibration of the intensity threshold value; numbers represent size in pixels.

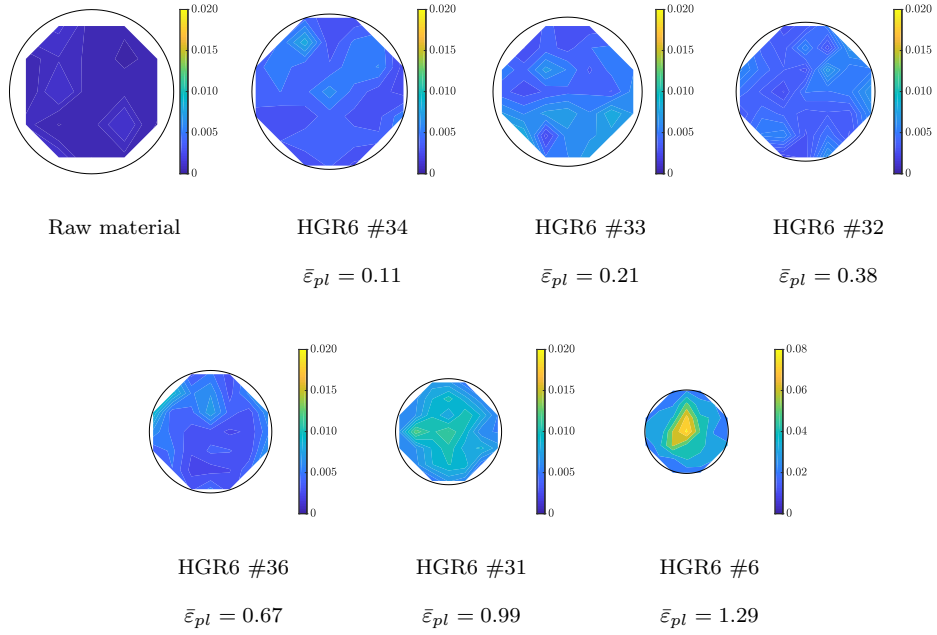


Figure 20: Contours of porosity in the cross-section of the analysed specimens.

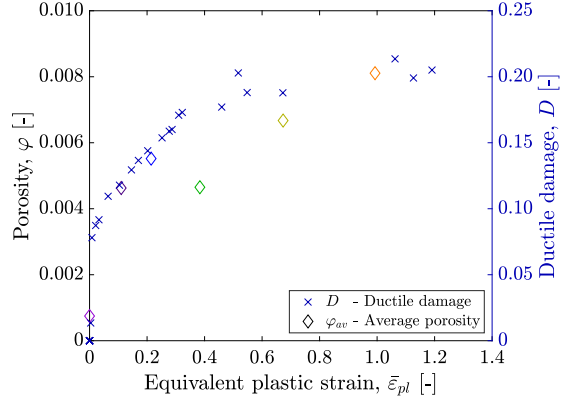


Figure 21: Analysis of the porosity and ductile damage evolution with equivalent plastic strain.

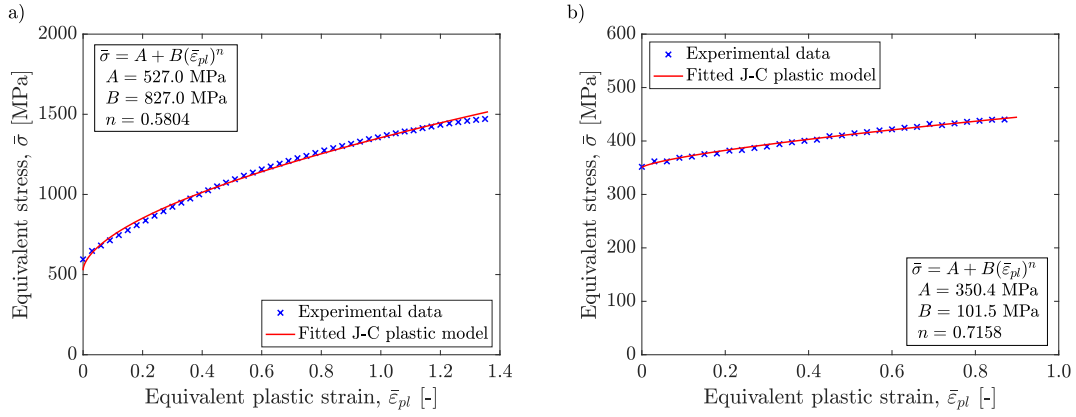


Figure 22: Experimental true equivalent stress - true equivalent plastic strain data fitted to Johnson-Cook plastic model: (a) stainless steel 304L; (b) copper C110.

Material	$A$ [MPa]	$B$ [MPa]	$n$
Stainless steel 304L	628.7	945.2	0.9314
Copper C110	354.1	259.2	0.9221

Table 6: Plastic model parameters.

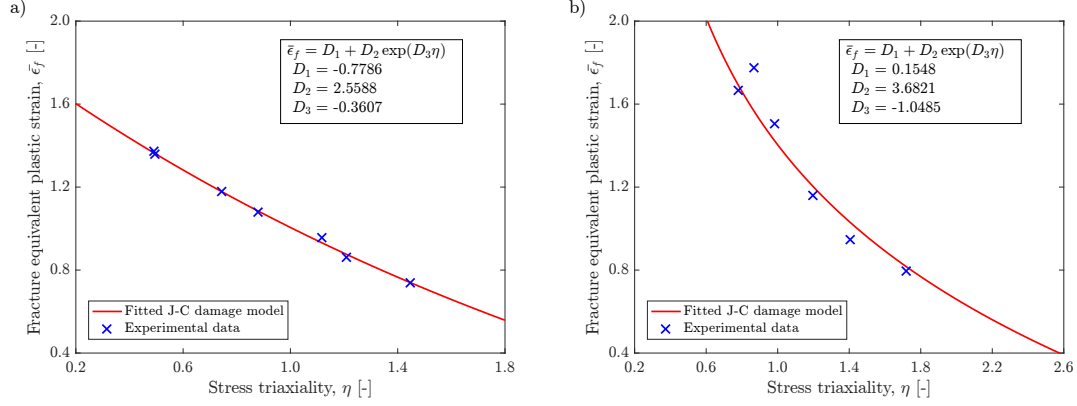


Figure 23: Experimental fracture equivalent plastic strain - stress triaxiality data fitted to Johnson-Cook fracture model: (a) stainless steel 304L; (b) copper C110.

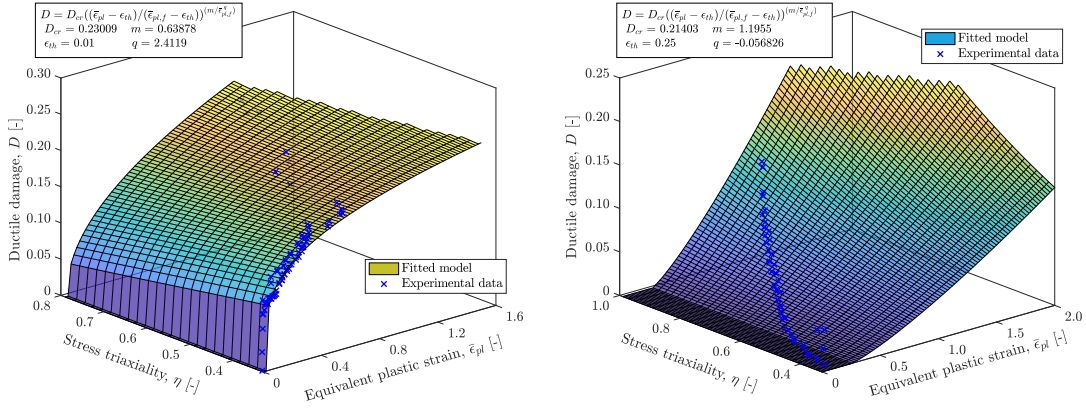


Figure 24: Experimental ductile damage as a function of equivalent plastic strain and stress triaxiality data fitted to the proposed damage accumulation model: (a) stainless steel 304L; (b) copper C110.

Material	$D_1$	$D_2$	$D_3$	$D_{cr}$	$\epsilon_{th}$	$m$	$q$
Stainless steel 304L	-0.7786	2.5588	-0.3607	0.226	0.01	0.4115	1.3058
Copper C110	0.1548	3.6821	-1.0485	0.214	0.25	1.1955	-0.0568

Table 7: Fracture and damage accumulation model parameters.